Comment on "T- dependence of the magnetic penetration depth in unconventional superconductors at low temperatures: Can it be linear?"

G.E. Volovik

Helsinki University of Technology, Low Temperature Laboratory, P.O.Box 2200, FIN-02015 HUT, Finland,
Landau Institute for Theoretical Physics, 117334 Moscow, Russia

(February 1, 2008)

PACS numbers: 74.25.Nf, 74.20.Fg, 74.72.Bk

In a recent Letter Schopohl and Dolgov (SD) suggested that a pure $d_{x^2-y^2}$-pairing state becomes invalid in the zero temperature limit, $T \rightarrow 0$.[1] Their arguments are based on thermodynamics: if the magnetic penetration length depends linearly on T at low T, the Nernst theorem – the third law of thermodynamics – is violated. We show here that this conclusion is the result of the incorrect procedure of imposing the limit $T \rightarrow 0$ in the electromagnetic response. To illustrate their reasoning let us consider a simplified case of the uncharged Fermi superfluid with lines of zeroes in the quasiparticle spectrum, the $d_{x^2-y^2}$-pairing being an example. In superfluids the density of the superfluid component $\rho_s(T)$ corresponds to the magnetic penetration length in superconductors, $1/\lambda^2(T) \propto \rho_s(T)$. In the case of the nodal lines it has linear dependence on T at low $T \ll T_c$: $\rho_s(T) = \rho - \rho_n(T)$, where the normal component density in such liquid is $\rho_n(T) \propto \rho T/T_c$. The kinetic energy contribution to the free energy of the liquid flowing with the superfluid velocity $\mathbf{v}_s$ along the channel is

$$\mathcal{F} = \frac{1}{2}\ \rho_s(T) v_s^2\ , \tag{1}$$

We consider the superflow circulating in an annular channel. This circulation is fixed, if one discards the negligibly small decay of the supercurrent via vortex formation, so one can consider $\mathbf{v}_s$ as temperature independent. This results in the finite entropy in $T = 0$ limit:

$$S(T=0) = -\frac{\partial \mathcal{F}}{\partial T}\bigg|_{T=0} = \frac{1}{2}\frac{\partial \rho_n}{\partial T}\bigg|_{T=0} v_s^2\ \propto\ v_s^2\ \frac{\rho}{T_c}\ . \tag{2}$$

If one follows the argumentation in Ref.[1], such a violation of the Nernst theorem suggests that the superfluid density ρ_s (or the related penetration length λ in superconductors) cannot be a linear function of T, which would mean that the pairing states with nodal lines are prohibited at $T = 0$ by the Nernst theorem.

There is however a loophole in this argumentation. The superfluid density $\rho_s(T)$ is the linear response function of the current $\mathbf{j}$ to the superfluid velocity $\mathbf{v}_s$, and thus is obtained in the limit $\mathbf{v}_s \rightarrow 0$. On the other hand the Nernst theorem requires the limit $T \rightarrow 0$ at finite $\mathbf{v}_s$. These two limits are not commuting for the kinetic energy $\mathcal{F}$. The crossover parameter, $x = T/p_F v_s$, regulates the scaling behavior of $\mathcal{F}$ in different limiting cases: $\mathcal{F}(T, x) = f(x)\rho v_s^2 T/T_c$, where $f(x)$ is dimensionless function of x.[2] The regime $x \gg 1$ corresponds to the linear response to the superfluid velocity, i.e. to the order of limits when $v_s \rightarrow 0$ first. In this 'high temperature' case, $T \gg p_F v_s$, the scaling function $f(x) \rightarrow$ Const and one obtains the finite entropy, $S(T) = \lim_{T\rightarrow 0} \lim_{v_s \rightarrow 0} -d\mathcal{F}/dT \propto \rho v_s^2/T_c$ in Eq. (2).

In the opposite limit of low T, $x \ll 1$, the scaling function has the asymptote $f(x) \rightarrow \frac{a}{x} + bx$, where a and b are parameters of order unity.[2] In this true Nernst limit the entropy is zero at $T = 0$:

$$\lim_{v_s\rightarrow 0}\lim_{T\rightarrow 0} -\frac{d\mathcal{F}}{dT} \propto\ v_s T\ \frac{\rho}{p_F T_c}\ , \tag{3}$$

in complete agreement with the Nernst theorem. Thus the linear T-dependence of the linear response function $\rho_s(T)$ does not violate the third law of thermodynamics: the Nernst principle does not prohibit a pure $d_{x^2-y^2}$-pairing state to exist at $T = 0$ in uncharged Fermi liquid.

The same can be immediately applied to the charged case, where the superfluid velocity $\mathbf{v}_s$ is to be substituted by the external electric current $\mathbf{j}$ discussed by SD.[1] Considering the true $T = 0$ limit of the energy, $\lim_{j\rightarrow 0} \lim_{T\rightarrow 0} -d\mathcal{F}/dT \propto jT$, one satisfies the Nernst principle. This does not contradict to the linear T-dependence of the linear electromagnetic response, which for the wave vector $k = 0$ gives

$$\lim_{T\rightarrow 0}\lim_{j\rightarrow 0} \frac{d\lambda(k=0, T)}{dT} =\ \mathrm{Const}\ . \tag{4}$$

For $k \neq 0$ there is another scaling parameter, $y = T/v_F k$, which regulates the dependence of the electromagnetic response on the wave vector k and produces the T^2 dependence of the penetration length at finite k, i.e. at $y \ll 1$.[3] In opposite case, $y \gg 1$, the Eq. (4) is restored.

In conclusion, lines of nodes in clean superconductors are not in conflict with Nernst theorem. The answer to the question in the title of their paper[1] is yes.

---

[1] N. Schopohl, O.V. Dolgov, Phys. Rev. Lett., **80**, 4761 (1998).
[2] G.E. Volovik, JETP Lett. **65**, 491 (1997).
[3] I. Kosztin, A.J. Leggett, Phys. Rev. Lett., **79**, 135 (1997).